Chapter 9
Model Tests in Ice

9-1. General

a. The need for physical modeling of a river hydraulic problem arises when the available analytical or numerical solution techniques are inadequate to describe either the processes involved or important details of those processes. Such instances may be ascribable to complex boundary conditions, multi-dimensional flow patterns, or imprecisely understood processes. Physical modeling may then provide a useful alternative, either by itself or in a hybrid approach using both numerical and physical modeling.

b. Small-scale laboratory modeling of hydraulic structures (locks, dams, weirs, spillways, etc.) and vessels under open-water conditions is now common, and the modeling laws, criteria, and techniques are well established (Yalin 1971, USDI 1980). The presence of ice brings serious complications to small-scale modeling because it adds a boundary at the top of the water body having surface characteristics that are different from those of the bed of the waterway. Moreover, whenever the mechanical properties of ice affect the problem under study, these must be reproduced in the model. The basic principle of dynamic similitude or modeling is to reproduce in the model the forces that govern the problem under consideration. The ratio between any two forces (gravity forces, inertia forces, viscous forces, shear forces, mechanical forces, etc.) should be the same in the model as in the prototype. Except for a few cases, all these forces usually play some role in the actual physical phenomena of interest. Thus, strict adherence to the principle of dynamic similitude will lead to the conclusion that the phenomena can only be studied at full scale. It then becomes necessary to relax the principle of similitude, and to choose to model exactly only those forces that primarily affect the problem under consideration. Simultaneously, the "scale effects," or errors introduced by imperfect modeling of the secondary forces, are held to a minimum by judicious model design. Therefore, it is important at the outset to correctly identify the primary forces that govern a particular phenomenon before attempting to study it in a physical model. This must be done to decide whether the necessary modeling techniques are available and how the model data can be extrapolated to full scale. In the present state of the art of ice modeling, phenomena that are strongly affected by heat transfer—e.g., refreezing of broken ice, icing of structures, and the like—are not amenable to physical modeling.

9-2. General Similitude Considerations

A model is said to be in complete similitude if it is geometrically, kinematically, and dynamically similar to the prototype. Geometric similarity requires that each linear dimension of the prototype, L_p, be equal to the corresponding model dimension, L_m, times the scale factor, λ_l:

$$L_p = \lambda_l \ L_m \ . \qquad (9\text{-}1)$$

a. Kinematic similarity requires the similarity of motion. This implies that corresponding velocities and paths of motion are similar. This is most commonly stated in terms of a velocity ratio, λ_v:

$$V_p = \lambda_v \, V_m \ . \tag{9-2}$$

For dynamic similarity, the ratio of corresponding masses and mass accelerating forces must be similar:

$$M_p = \lambda_m \, M_m \tag{9-3a}$$

$$F_p = \lambda_f \, F_m \ . \tag{9-3b}$$

b. Turbulent open channel flows are driven by the force of gravity and resisted by friction. This leads to at least three types of forces that must be accounted for in such flows: gravity, friction, and inertia. In terms of force ratios, this requirement suggests that both Froude and Reynolds scaling must be obeyed. To satisfy more than one model law in a physical model, however, typically requires the ability to vary the properties of the testing fluid over a relatively broad range. Since it is generally not practicable to use a fluid other than water in a physical model, similitude in models of river hydraulics is generally based on Froude scaling, while trying to minimize viscous effects (Reynolds number independence) by maintaining a high enough Reynolds number to ensure fully rough turbulent flow.

c. For free surface flows, the transition from laminar to turbulent flow occurs between Reynolds numbers (based on hydraulic radius) of 500 to 2000. In this transition range, resistance to flow shifts from a primary dependence on viscous shear to a dependence on the relative roughness of the channel or form drag. For physical models of free surface flow, the viscous effects have often been minimized by maintaining a Reynolds number, based on hydraulic radius, of 1400 to 2500. An alternate criterion that is often employed is the roughness Reynolds number:

$$\mathrm{Re}^* = V^* \, k/\upsilon > 100 \tag{9-4}$$

where V^* is the shear velocity, k is a composite hydraulic roughness corresponding to an equivalent sand grain roughness, and υ is the kinematic viscosity of water. In the case of river ice models, an additional constraint is imposed, since the addition of a continuous, stationary ice cover to a wide channel effectively doubles the wetted perimeter and halves the hydraulic radius. In that case, the Reynolds number criterion must likewise be increased.

d. In addition to gravitational and viscous forces, surface tension must also be considered in the design of river ice models. The importance of surface tension can be reflected through the Weber number, which is a ratio between surface energy forces and inertia. As in the case of the Froude number, the Weber number deals with the interface between two fluids (or in some cases a fluid and a solid boundary), but like the Reynolds number it is of primary significance in flows of small depths and velocities. Surface tension effects are typically overcome by providing sufficient flow depths in areas of interest, but alternatives include controlling the surface texture of the boundaries or chemical additives to the fluid.

e. When sheet ice failure is to be modeled, such as in ice–structure interaction studies—e.g., ice forces on piers or icebreaker model tests—the ice's relevant physical and mechanical properties, such as thickness h, density ρ_i, friction factor f_i, flexural strength σ_f, crushing strength σ_c, and elastic modulus E, must be properly modeled. The ice's mechanical properties at prototype and model scales are related by:

$$\sigma_p = \lambda_\sigma \sigma_m \tag{9-5a}$$

$$E_p = \lambda_E E_m \tag{9-5b}$$

$$(\rho_i)_P = (\rho_i)_M \tag{9-5c}$$

$$(f_i)_P = (f_i)_M \tag{9-5d}$$

so that the following relationships are satisfied

$$(E/\sigma)_p = (E/\sigma)_m \tag{9-5e}$$

$$(\sigma/\rho g h)_p = (\sigma/\rho g h)_m \tag{9-5f}$$

where g is gravity.

f. Similitude relationships for river ice modeling have been presented by numerous authors (e.g., Ashton 1986). Since these texts and references and reports are widely available and in general agreement, their development will not be repeated here. The scaling ratios are presented in Table 9-1, which adopts the convention that the ratios are prototype values divided by the model values so that the geometric length scales are always greater than one. Also, for the sake of simplicity, the density ratios of the water and the ice modeling material are assumed to be unity. More details can be found in Ashton (1986).

9-3. Undistorted Models

Undistorted models are those in which all geometric lengths are scaled by the same ratio, and the first several items in Table 9-1 are simply scaled as products of the geometric length ratio. Since the significant processes in most open channel flow problems are dominated by the forces of gravity and inertia, the remaining ratios are the consequence of requiring that the Froude numbers of the model and prototype be equal:

$$\{V/(g\,D)^{0.5}\}_p = \{V/(g\,D)^{0.5}\}_m \tag{9-6}$$

where V and D are the average flow velocity and flow depth, respectively. Since gravitational acceleration is relatively constant, Equation 9-6 reduces to:

$$\lambda_v = V_p/V_m = (D_p/D_m)^{0.5} = \lambda^{0.5}\ . \tag{9-7}$$

Table 9-1.
Scaling Laws for Ice Physical Models

Variable	Symbol	Scaling ratio: Distorted	Scaling ratio: Undistorted ($\beta=\lambda$)
Length, horizontal	X	λ	λ
Length, vertical	Z	β	λ
Area, horizontal	A_x	λ^2	λ^2
Area, vertical	A_z	$\lambda\beta$	λ^2
Volume, Mass	$\forall$, M	$\lambda^2\beta$	λ^3
Velocity	V	$\beta^{1/2}$	$\lambda^{1/2}$
Discharge, horizontal	Q_x	$\lambda\beta^{3/2}$	$\lambda^{5/2}$
Discharge, vertical	Q_z	$\lambda^2\beta^{1/2}$	$\lambda^{5/2}$
Time, horizontal	T_x	$\lambda/\beta^{1/2}$	$\lambda^{1/2}$
Time, vertical	T_z	$\beta^{1/2}$	$\lambda^{1/2}$
Acceleration, horizontal	a_x	β/λ	1
Acceleration, vertical	a_z	1	1
Force, horizontal	F_x	$\lambda\beta^2$	λ^3
Force, vertical	F_z	$\lambda^2\beta$	λ^3
Ice strength, compressive	σ_c	β	λ
Ice strength, flexural	σ_f	λ^2/β	λ
Ice strength, shear (vertical)	σ_s	λ	λ
Modulus of Elasticity			
Flexure	E_f	λ^4/β^3	λ
Compression	E_c	β	λ
Buckling	E_b	β	λ

a. For dynamic similarity, forces must also be appropriately scaled. Since Froude scaling is based on keeping the ratios of inertial to gravity forces constant from model to prototype:

$$\{F_i/F_g\}_p = \{F_i/F_g\}_m \tag{9-8a}$$

or

$$\rho_p/\rho_m \lambda^3 a_p/a_m = \rho_p/\rho_m \lambda^3 g_p/g_m \tag{9-8b}$$

where a is the acceleration. Because we cannot effectively vary gravity, Equation 9-8b reduces to:

$$a_p/a_m = g_p/g_m = 1 \tag{9-8c}$$

so that the force ratio may be expressed as

$$F_r = F_p/F_m = \lambda^3. \tag{9-9}$$

b. The stress ratios that follow in Table 9-1 are a direct consequence of applying a force, scaled as λ^3, over an area, λ^2. We can also examine the roughness required in the model by referring to the Darcy-Weisbach equation for the frictional head loss H_f in a pipe of length L and diameter D:

$$H_f = f\,(L/D)\,(V^2/2g) \quad . \tag{9-10}$$

Substituting $4R_h$ (hydraulic radius) for the pipe diameter, slope S for H_f/L, rearranging and taking the ratio of Equation 9-10 for prototype and model conditions gives the following expression for the slope ratio S_r in terms of the Froude number ratio Fr:

$$S_r = f_r\,\mathrm{Fr}_r^2 \; . \tag{9-11a}$$

Similarly, using the Manning equation it can be shown that

$$S_r = \{n_r^2/R_h^{1/3}\}\ \mathrm{Fr}^2 \tag{9-11b}$$

where n is Manning's roughness coefficient and R_h is the hydraulic radius. For a wide channel R_h is approximately equal to the depth D.

c. Because the modeling is based on equality of Froude numbers for model and prototype (Fr_r = 1), and since the slope ratio in an undistorted model is also unity, the boundary friction factor should be equal in model and prototype. If the model is designed to operate under Reynolds number independence, then the friction factor should be governed primarily by the relative roughness of the boundary surface, k/R_h (where k is the composite roughness of the channel). If the model flow is in the transitional regime where boundary resistance varies with Reynolds number, it may be necessary to distort the relative roughness empirically. Similar reasoning indicates that the friction between the ice and solid boundaries or other ice pieces should also be the same in model and prototype.

9-4. Distorted Models

While undistorted models, i.e., models with the same scale in both the horizontal and vertical directions, are by far preferable, distorted hydraulic models may have to be used when modeling long reaches of wide rivers. This is accomplished by exaggerating the vertical scale relative to the horizontal scale. In some cases, there may be different scaling ratios for all three geometric lengths, or there may be a distortion of the channel slope that is distinct from the length scale ratios (tilting). In the case of river ice models, a separate length scale for ice thickness, distinct from the vertical length scale, has also been proposed (Michel 1975).

a. The need for a distorted model may also arise from limitations on the available space in which to construct the model, or because of a lack of control over the modeling materials and conditions. The size of the modeling facility often limits model scales because most water bodies are relatively shallow in comparison to their plan dimensions. If the prototype area to be modeled is large, the scale reductions necessary to fit the model within the available (or economically feasible) space may be so great that vertical dimensions cannot be measured with adequate resolution, or viscous and surface tension effects become dominant. In this case, the vertical scale may be distorted relative to the horizontal scale provided that appropriate modifications are made to the remaining scale ratios, as shown in Table 9-1.

b. The second reason for distorted models arises from the limitations on modeling materials and conditions. As mentioned previously, for large-scale physical models, it is rarely practical to use a model fluid other than water, and gravitational forces are essentially equal in model and prototype. For cases where the mechanical properties of ice are important, the practical limitations of available artificial ice materials have limited undistorted models to geometric scale ratios no greater than 20 (Michel 1978). Timco (1979) reported that doped real ice may be used at scales up to 50 (see Paragraph 9-5*b*).

c. While model distortion is often required because of either scaling or economics, it must be recognized that the distortion leads to improper conversions between potential and kinetic energy. Distortion is reasonable for flows that are largely two dimensional and that exhibit essentially hydrostatic pressure distributions. If flows have significant three-dimensional qualities, however, distortion presents significant limitations since vertical accelerations are not properly reproduced. In general, distorted models are not well suited to situations where there is significant curvature of the water surface.

d. When a model is distorted, the distortion must be planned to accomplish a specific goal (such as the prediction of water levels); however, the replication of other model characteristics (such as velocity distributions or forces on structures) may be significantly impaired despite the use of distorted scaling ratios. Scaling ratios for even basic quantities such as the hydraulic radius and boundary roughness become cross-section- or reach-specific, since they depend on cross-sectional shape. For small particles, their interaction may not distort items such as the angle of repose or internal friction. What does and does not distort properly is not always clear.

e. The scaling ratios for the case of a distorted model can be developed in a fashion similar to that used above for the undistorted case, but the task is far more complex. A detailed review of distortion in river ice models is beyond the scope of this chapter, and the reader is referred to other texts such as Michel (1978), Ashton (1986), and Wuebben (1995). If properly applied, distorted Froude scaling will ensure proper ratios of inertia to gravity forces taken separately in the vertical and horizontal directions. However, numerous scale effects will arise owing to dilatation resulting from forces scaling by different ratios in the vertical and horizontal directions. A literature review of previous river ice models has shown that distortions have typically been limited to four or fewer (Wuebben 1995).

9-5. Model Ice Materials

a. Modeling Broken Ice. In phenomena that do not involve a solid ice sheet but only ice floes or brash ice, the main forces to consider are usually gravity forces, but they also may include buoyancy forces, inertia forces, and possibly shear forces ascribable to water flowing underneath the stationary floes (e.g., ice held at a retaining structure such as an ice boom). If ice-on-ice friction is not thought to be critical, artificial ice floes can be used instead of real ice floes in the model, as long as the density of the material is equal to that of ice (e.g., polyethylene). The model study can then be made in an unrefrigerated facility with significantly reduced costs.

Several types of model brash ice have been used, usually made of pieces or crushed plastic material of a density equal or very close to that of real ice (Zufelt and Ettema 1996). An example of a model study using this type of material is found in Larsen et al. (1994) and shown in Figure 9-1.

Figure 9-1. Model of Niagara River using crushed polyethylene as model ice.

b. Modeling Sheet Ice. When the phenomenon to be studied involves the failure or breaking of an initially intact ice cover (e.g., ice forces on structures), or the secondary breaking of large floes, the mechanical properties of ice (bending strength, crushing strength, shear strength, and ice friction) become important and must be properly modeled in the laboratory. To achieve correct mechanical properties, most model ice is grown from a solution of salt, urea, glycol, or some other dopant in water. The achievable model properties, however, impose a minimum limit to the model scale. This limiting scale will depend upon the mode of failure of the ice sheet (e.g., the limiting scale is approximately 1:40 for ice failing in bending). A refrigerated facility is necessary for this type of modeling and a discussion of such a model study is given in Gooch and Deck (1990). Figure 9-2 shows a model using real ice in the CRREL refrigerated model area. Some artificial materials have been developed that are claimed to reproduce the properties of real ice, but their composition is proprietary, their handling is often messy, and even though they can be used in a warm environment, the cost of the experiments is similar to those in refrigerated facilities.

Figure 9-2. Model of ice jam control structure for Cazenovia Creek, New York.

9-6. Model Calibration

Once a modeling technique has been chosen and the physical model built, it should be calibrated or verified. This process usually consists of the following steps: adjustment of bed roughness to reproduce the water surface profile without ice (this is the normal model verification for conventional hydraulic models); verification of head losses with a simulated ice cover for known field conditions; and verification of the similitude of ice processes for known field conditions, such as ice breakup, ice drift pattern, and velocity. Even if this last verification is only qualitative, it is necessary to ascertain that the model is simulating observed natural phenomena. The objective of the calibration of a hydraulic model is to reproduce more or less normal field conditions, so that the model can be used to predict the effects of *abnormal* conditions or those produced by man-made changes with a good degree of confidence. In an ice–hydraulic model, it is not sufficient to reproduce water levels at various discharges as in a conventional hydraulic model. The ice phenomena also have to be correctly simulated. Many ice phenomena are not fully understood. If they are not carefully observed and documented at the particular field site to be modeled, it is unlikely that they can be simulated correctly in the model.

9-7. Considerations in Choosing Modeling

While proper physical hydraulic modeling must follow some basic scientific and engineering principles, it still remains as much an art as a science. This is even more true when ice effects are involved. In this regard, the experience of the engineer in charge of a model study is critical to the success of the study and to the reliability of its results. Physical modeling can be a very powerful tool in deciding among various potential designs for a project or among proposed solutions to a particular problem, in optimizing an initial design, in providing rational answers to objections to a proposed design or project, and in detecting potentially undesirable effects of a proposed design or solution, which may not have been foreseen otherwise, or not predicted by numerical modeling. While a physical model study often is a costly endeavor, when properly conducted, it can point the way to design or construction savings that often will more than offset its cost (Figure 9-3).

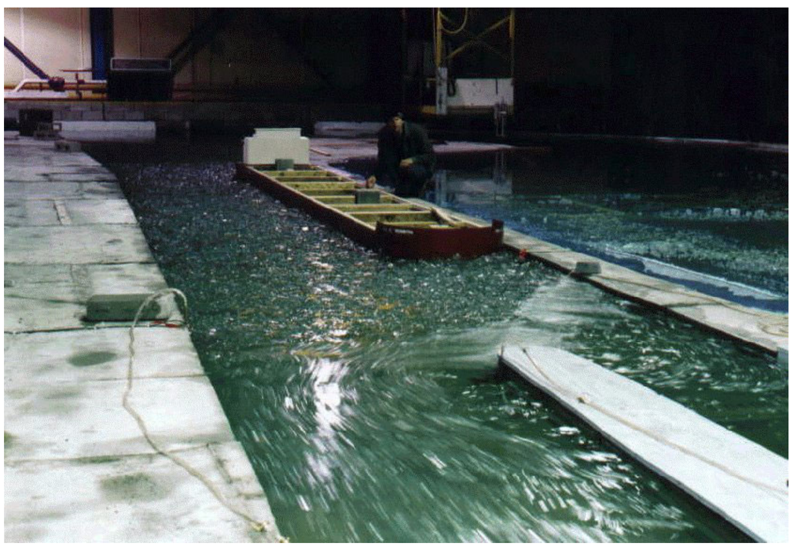

Figure 9-3. Model of Soo Locks, Michigan—ice control in upper lock approach.

9-8. References

a. Required publications.

None.

b. Related publications.

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